

- KIKUCHI, Y., HIROTA, E. & MORINO, Y. (1961). *Bull. Chem. Soc. Japan*, **34**, 348.
- KOJIMA, S., TSUKADA, K., OGAWA, S. & SHIMAUCHI, A. (1953). *J. Chem. Phys.* **21**, 2237.
- MORINO, Y., MIYAGAWA, I., CHIBA, T. & SHIMOZAWA, T. (1956). *J. Chem. Phys.* **25**, 185.
- MORINO, Y., CHIBA, T., SHIMOZAWA, T. & TOYAMA, M. (1958). *J. Phys. Soc. Japan*, **13**, 869.
- MORINO, Y., CHIBA, T., SHIMOZAWA, T., TOYAMA, M. & ITO, K. (1959). *Rev. Univ. des Mines*, 9e Ser. T. XV, 591.
- MORINO, Y. & TOYAMA, M. (1960). *J. Phys. Soc. Japan*, **15**, 288.
- MORINO, Y. & TOYAMA, M. (1962). *J. Chem. Phys.* **35**, 1289.
- ROBINSON, H., DEHMELT, H. G. & GORDY, W. (1954). *J. Chem. Phys.* **22**, 511.
- STERNHEIMER, R. M. (1954). *Phys. Rev.* **95**, 736.
- TOYAMA, M. (1959). *J. Phys. Soc. Japan*, **14**, 1727.
- VAN BOMMEL, A. J., STRIJK, B. & BIJVOET, J. M. (1950). *Proc. Koninkl. Nederland Akad. Wetenschap*, **53**, 47.

*Acta Cryst.* (1963). **16**, 134

## The Synthesis and Crystal Structures of some Alkaline Earth Titanium and Zirconium Sulfides

BY ABRAHAM CLEARFIELD

*National Lead Company, Titanium Alloy Division, Niagara Falls, New York, U.S.A.*

(Received 26 July 1961 and in revised form 13 April 1962)

Barium titanium sulfide ( $\text{BaTiS}_3$ ) and calcium, strontium and barium zirconium sulfides ( $\text{CaZrS}_3$ ,  $\text{SrZrS}_3$  and  $\text{BaZrS}_3$ ) have been synthesized by the reaction of  $\text{CS}_2$  with the respective ternary oxides at elevated temperatures. Barium titanium sulfide is hexagonal but its sulfur content decreases with increasing temperature of preparation. This results in a variation of unit cell dimensions from  $a = 6.752$ ,  $c = 5.810$  Å when prepared at 700 °C. to  $a = 6.77$ ,  $c = 5.74$  Å when prepared at 1100 °C.  $\text{BaTiS}_3$  containing a stoichiometric or near stoichiometric equivalence of sulfur has the barium nickelate structure. As the sulfur content decreases the structure becomes increasingly disordered.  $\text{BaZrS}_3$  is orthorhombic with  $a = 7.037$ ,  $b = 9.983$ ,  $c = 7.050$  Å and has a distorted perovskite structure.  $\text{SrZrS}_3$  and  $\text{CaZrS}_3$  have even more highly distorted perovskite structures and require orthorhombic unit cells with  $a = 13.49$ ,  $b = 9.79$ ,  $c = 14.23$  Å for  $\text{SrZrS}_3$  and  $a = 13.07$ ,  $b = 9.58$ ,  $c = 14.05$  Å for  $\text{CaZrS}_3$ .

### Introduction

Hahn & Mutschke (1956) prepared  $\text{SrTiS}_3$ ,  $\text{BaTiS}_3$  and  $\text{BaZrS}_3$  by heating mixtures of the binary sulfides in sealed evacuated tubes for several weeks. The strontium and barium titanium sulfides were isomorphous and could be indexed by means of hexagonal, orthorhombic or tetragonal unit cells. These compounds were tentatively assigned the hexagonal  $\text{BaNiO}_3$  structure (Lander, 1951). However, some serious discrepancies between the calculated and observed intensities were obtained. Barium zirconium sulfide was found to be dimorphic. The allotrope prepared below about 800 °C. was found to be tetragonal with  $a = 4.990$ ,  $c = 5.088$  Å and to have a perovskite structure. The high temperature form could not be indexed.

This paper describes the preparation of alkaline earth titanium and zirconium sulfides by the action of  $\text{CS}_2$  on the corresponding ternary oxides together with additional observations on their structure.

### Experimental

All of the titanates and zirconates used were c.p. grade (TAM Division, National Lead Company) and

contained less than 0.2% total metallic impurities. Powder patterns of the ternary oxides showed the presence of only the desired phase. The sulfiding procedure was similar to that used for the preparation of  $\text{ZrS}_2$  (Clearfield, 1958). Argon was bubbled through a tower containing  $\text{CS}_2$  and the resultant gas mixture then led into a zircon combustion tube containing 10 g. of the ternary oxide in a carbon boat. The flow rate of the argon was fairly constant so that the amount of  $\text{CS}_2$  entering the combustion tube was roughly proportional to the reaction time. Powder patterns were obtained with a G. E. XRD-3 X-ray unit and diffractometer using filtered  $\text{Cu } K\alpha$  radiation. Aluminum powder and aluminum oxide were used as internal standards for the determination of accurate unit cell dimensions.

### Barium titanium sulfide

The preparation of  $\text{BaTiS}_3$  was carried out at one hundred degree intervals in the temperature range 400–1100 °C. Only small amounts (up to 30%) of sulfide were obtained in 4 hours time below 700 °C. However, the products obtained in 4 hours reaction

time at 700 to 1000 °C. showed only the presence of the single phase BaTiS<sub>3</sub>. The powder patterns of the 1100 °C. preparations contained some new reflections. Reaction times in the order of 24 hours were required to eliminate or reduce to low values the intensities of these reflections.

The powder patterns could be indexed on the basis of either an orthorhombic or hexagonal unit cell. However, the unit cell dimensions changed slightly with temperature of preparation. This is shown in Fig. 1 by the increased resolution of the (200), (002) doublet (hexagonal indexing) as a function of temperature of preparation. In all of the BaTiS<sub>3</sub> samples the *a* axis of the orthorhombic unit cell equaled  $\sqrt{3}c$  and the *a*-*c* diagonal equaled  $2c$  and made an angle of 60° with the *c* axis. Thus, the true unit cells are most probably hexagonal with the dimensions given in Table 1. The values reported by Hahn & Mutschke are included for comparison.

Table 1. Hexagonal unit-cell dimensions of BaTiS<sub>3</sub>

Sample no.	Temp. of preparation	<i>a</i> (Å)	<i>c</i> (Å)
2	700 °C.	6.752 ± 0.006	5.810 ± 0.005
3	1000	6.758 ± 0.008	5.778 ± 0.006
5	1100	6.77 ± 0.01	5.74 ± 0.01
Hahn & Mutschke		6.730	5.829

Analytical data for a number of BaTiS<sub>3</sub> preparations are given in Table 2. Each of the products also contained 0.1 to 0.3% insolubles (carbon, silica). The fact that the elemental composition of each sample totaled less than 99.7% indicates that oxygen was incompletely replaced by sulfur. Also, the sulfur deficiency increased with increasing temperature of preparation. Thus, the variation in unit cell dimensions appears to be related to the sulfur content of the products.

The unit cell volumes of all the preparations listed in Table 1 are practically identical and contain 2 molecules of BaTiS<sub>3</sub> based on Hahn & Mutschke's observed density of 4.00 g.cm.<sup>-3</sup>. Integrated intensities were obtained for preparations No. 2 (700°) and No. 3 (1000°) by counting the area under the peaks for the first 6 reflections using the scaler. The intensities for the remaining reflections were estimated by comparison with the measured reflections. Intensity calculations were first made assuming the barium nickelate structure, space group  $C_{6v}^4-P6_3mc$ ,

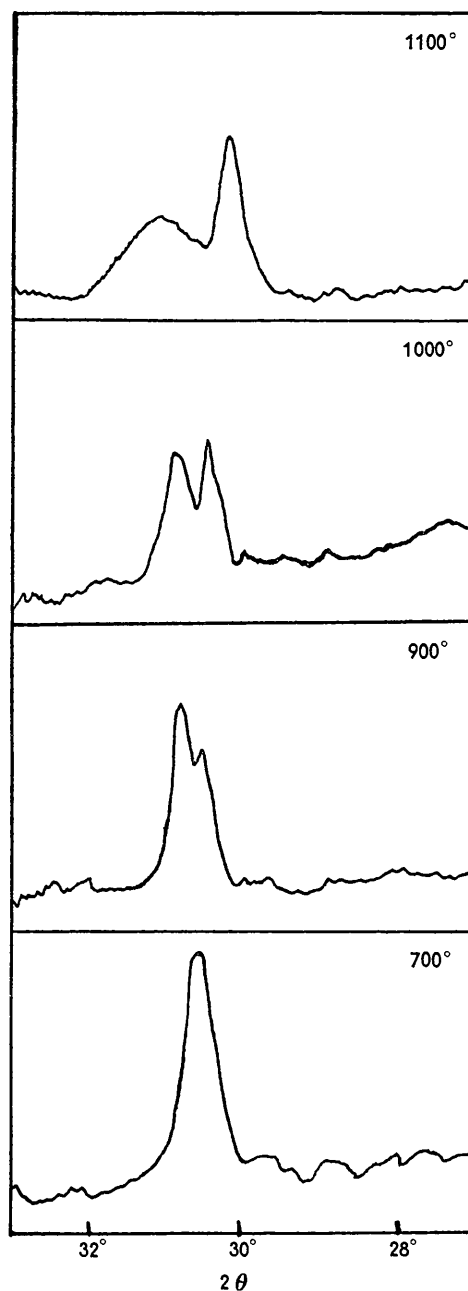


Fig. 1. Effect of temp. of preparation of BaTiS<sub>3</sub> upon the intensity and resolution of (200), (002) doublet.

Table 2. Analytical data for BaTiS<sub>3</sub>

Sample no.	Reaction temp. (°C.)	Reaction time (hr.)	% Ba	% Ti	% S	S/Ti	S/Ba
1	700	4	—	—	33.5	—	—
2	700	24	48.25	17.01	34.4	3.02	3.06
3	1000	4	47.82	17.22	33.46	2.90	3.00
4	1000	4	48.65	17.12	33.90	2.96	2.98
5	1100	24	49.78	16.81	32.89	2.88	2.79
	Required for BaTiS <sub>3</sub>		48.80	17.02	34.18	3.00	3.00

Table 3. Structure data for BaTiS<sub>3</sub> prepared at 700 °C.

<i>hkl</i>	<i>d</i> <sub>o</sub>	<i>d</i> <sub>c</sub>	<i>I</i> <sub>c</sub> × 10 <sup>-4</sup>	Σ <i>I</i> <sub>c</sub> × 10 <sup>-4</sup>	<i>I</i> <sub>o</sub> × 10 <sup>-4</sup>	<i>hkl</i>	<i>d</i> <sub>o</sub>	<i>d</i> <sub>c</sub>	<i>I</i> <sub>c</sub> × 10 <sup>-4</sup>	Σ <i>I</i> <sub>c</sub> × 10 <sup>-4</sup>	<i>I</i> <sub>o</sub> × 10 <sup>-4</sup>				
100	—	5·848	0·5	0·5	0	511	—	1·034	3·0						
001	—	5·810	0	0	0	422	1·034 <i>B</i>	1·033	9·0	15·9	16·6				
101	4·128	4·122	244	244	237	404		1·030	1·0						
110	3·387	3·377	210	210	215	215		1·029	2·9						
111	2·924	2·920	0	0	110	503	—	1·001	1·6	1·6	0				
200		2·924	39·2	123	473	305	—	0·9981	0	0	0				
002		2·906	2·905			84	512	—	0·9878	2·7	2·7	0			
201	2·614	2·612	415			324	—	0·9855	0·05	0·05	0				
102	2·201	2·602	84	499	473	600	0·9749	0·9744	5·6	5·6	9·0				
210		2·210	0·4			333	—	0·9731	0	0	0	0			
112		2·202	19·5			19·9	006	—	0·9683	0·9	0·9	0			
211	2·063	2·066	68	172	161	430	—	0·9614	0·1	0·1	0				
202		2·061	104			601	—	0·9613	0	0	0	0			
300		1·949	1·949			37	37	423	0·9594 <i>B</i>	0·9599	12·5	18·1	14·6		
003	—	1·937	0	0	414	0·9587	5·6	5·6		0					
301	—	1·848	0	0	225	—	0·9572	0		0	0				
103	1·836	1·839	22·4	22·4	19	106	—	0·9553	1·3	1·3	0				
212	1·758	1·759	43	43	30	431	—	0·9485	2·4	2·4	0				
113	—	1·680	0	0	0	315	—	0·9446	2·4	2·4	0				
220	1·688	1·688	67	67	77	520	0·9368	0·9364	2·9	2·9	4				
310	—	1·622	0·2	0·2	0	116	—	0·9308	0·6	0	0				
221		1·621	0			521	—	0·9246	0	0	0	0			
302		1·619	6·1			65·6	57	602	0·9237 <i>B</i>	0·9241	4·8	7·0	8·5		
203	1·613	1·615	59·5	513	0·9233			2·2							
311	1·565	1·562	21	21	206			—		0·9192	3·3			3·3	0
400	1·459	1·462	2·4	61·9	54	432	—	0·9127	0·5	0·5	0				
222		1·460	33·3			504	—	0·9110	0	0	0	0			
213		1·457	14·9			405	—	0·9097	5·8	5·8	5·8	0			
004	1·417	1·452	11·3	45·8	41·4	610	—	0·8918	0·02	0·02	0				
401		1·418	31·8			522	—	0·8913	1·1	1·1	0	0			
312		1·416	14·0			334	—	0·8896	2·7	2·7	0	0			
104	—	1·410	0·1	0·1	0	216	—	0·8866	2·2	2·2	0				
303	—	1·374	0	0	0	611	—	0·8815	2·0	2·0	0				
320	—	1·342	0·1	0·1	0	424	—	0·8795	1·4	1·4	0				
114	1·333	1·334	12·2	12·2	5·8	325	—	0·8783	2·1	2·1	0				
321	1·307 <i>B</i>	1·307	8·9	25	26	603	—	0·8703	0	0	0				
402		1·306	13·2			306	—	0·8672	0·02	0·02	0	0			
204		1·302	2·9			433	—	0·8611	2·1	2·1	0	0			
410	1·277	1·276	10	10	13	415	—	0·8592	0	0	0				
223	—	1·273	0	0	0	612	—	0·8526	2·1	2·1	0				
411	1·244	1·247	0	6·7	5·2	514	—	0·8511	0·02	0·02	0				
313		1·244	6·7			440	0·8445	0·8441	4·5	4·5	3·5	0			
322		1·217	1·218			6·7	523	—	0·8430	0	0	0			
214	1·168 <i>B</i>	1·214	0·1	6·8	5·2	226	—	0·8399	4·6	4·6	0				
500		1·170	0			530	—	0·8354	0·01	0·01	0	0			
412		1·168	2·5			21·6	22·8	700	—	0·8353	0·01	0·02	0		
403	1·167	12·7	441	—	0·8353			0	0	0	0				
304	1·165	6·4	316	—	0·8314			2·3	2·3	0	0				
005	—	1·162	0	0	0	007	—	0·8300	0	0	0				
501	1·147	1·147	2·3	4·6	4	531	—	0·8269	2·2	3·3	0				
105		1·139	1·140			2·3	701	—	0·8269			1·1			
330		1·125	1·125			2·7	2·7	3·5	505			—	0·8243	1·1	1·1
420	1·102 <i>B</i>	1·105	1·4	23·2	17	107	—	0·8218	1·0	1·0	0				
331		1·105	0			620	—	0·8110	0·7	0·7	0	0			
323		1·103	4·0			442	—	0·8106	4·9	4·9	0	0			
224	1·086 <i>B</i>	1·101	17·8	29·6	41	613	0·8095 <i>B</i>	0·8101	2·4	21·9	22·7				
115		1·099	0			604		—	0·8094			10·3	10·3	0	0
421		1·086	18·4			335		—	0·8084			0	0	0	0
502	1·086	1·086	1·9	29·6	41	406	—	0·8073	3·6	0	0				
314		1·082	0·1			117	—	0·8060	0			0	0	0	
205		1·080	9·2			621	—	0·8034	11·7			11·7	0	0	
413	—	1·066	0	0	0	702	—	0·8029	1·3	30·6	38·3				
510	—	1·050	0	0·8	0	532	0·8036 <i>B</i>	0·8029	1·2						
332	—	1·049	0·8	0	0	434		0·8017	0						
						425		0·8008	11·1						
						207	—	0·7985	5·3						

and the same atomic positions chosen by Hahn & Mutschke (1956). This calculation confirmed the discrepancies noted earlier by these authors. However,

when the barium atom parameters were changed to  $\frac{2}{3}, \frac{1}{3}, \frac{1}{4}; \frac{1}{3}, \frac{2}{3}, \frac{3}{4}$  the discrepancies disappeared.

Plots of  $\log I_o/I_c$  versus  $(\sin \theta/\lambda)^2$  were made for

both the 700° data and the 1000° data. Intensities were then calculated from the equations

$$I_c = m \times LP \times |F_c|^2 \exp -2B (\sin \theta / \lambda)^2$$

$$I_c = S \times I_o$$

where

$m$  = multiplicity  
 $LP$  = Lorentz and polarization factor  
 $S$  = scale factor.

The values of  $S$  and  $B$  were  $4.73 \times 10^4$  and  $2.61$  for sample No. 2 (700° data) and  $4.35 \times 10^4$  and  $3.53$  for sample No. 3 (1000° data). A comparison of the observed and calculated intensities for sample No. 2 is given in Table 3. The reliability index,  $R = \Sigma(|I_o - I_c|) / \Sigma I_o$ , was 0.11 for the 700° data and 0.16 for the 1000° data.  $R$  was calculated using half the least observable value of  $I_o$  for the unobserved reflections.

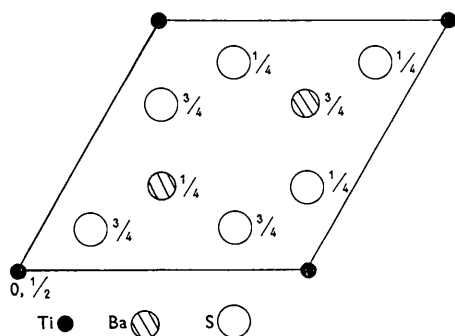


Fig. 2. Unit cell of  $BaTiS_3$  projected onto (001) plane.

A projection of the unit cell onto the (001) plane is shown in Fig. 2. Each titanium atom is octahedrally

coordinated by sulfur atoms. Each barium atom is surrounded by 12 sulfur atoms in a hexagonal close packed arrangement. The interatomic distances for the 700° preparation are given in Table 4. The primed sulfur atoms refer to those lying in planes above and below the barium atom while the unprimed ones lie in the same plane as the barium atom.

Table 4. *Interatomic distances in  $BaTiS_3$*

6 Ti-S	2.43 Å
6 Ba-S	3.38
6 Ba-S <sup>1</sup>	3.49

Table 5. *Variation in intensities of  $BaTiS_3$  with temperature of preparation*

$hkl$	$I_o$		
	Prep. no. 2 (700°)	Prep. no. 3 (1000°)	Prep. no. 5 (1100°)
101	237	234	207B
200	44	42	38
002	66	45	42B
201	473	430	398
102			
103	19	26	33
220	77	75	64
302	57	50	34B
203			
114	5.8B	—	—
412	22.8B	10B	6B
403			
304			

$B$  denotes broadened reflections.

The reason for the poorer agreement obtained with the higher temperature data became apparent upon examination of the differences in the powder patterns of the several preparations. Many reflections for which  $l$  is not zero become broadened and undergo large

Table 6. *Products obtained on reacting  $BaZrO_3$  with  $CS_2$  at several different temperatures*

Reaction temp. (°C.)	Reaction time (hr.)	Color of product	Sulfur content (%)	% Composition of product		
				$BaZrO_3$	$BaZrS_3$	Unknown phase
500	4	light orange	—	100	—	—
500	12	orange	—	85	15	0
600	4	orange	2.45	100	0	0
600	12	red	7.05	80	20	0
700	4	orange	4.18	88	12	0
750	4	red	9.65	68	25	7
750	18	violet	—	15	75	10
800	4	maroon	15.3	40	40	15
900	4	violet	21.18	20	70	10
1000	4	grey-black	27.10	0	100	0
1050	4	blue-black	28.60	0	100	0
1050	24	blue-black	29.60	0	100	0
Required for $BaZrS_3$			29.52			

Table 7. *Analytical data for alkaline earth zirconium sulfides*

Compound	Reaction temp. (°C.)	Reaction time (hr.)	Color	$A^{2+}$	Found			Required	
					Zr	S	$A^{2+}$	Zr	S
$BaZrS_3$	1050	4	Blue-black	42.10	28.10	28.60	42.16	28.32	29.52
$SrZrS_3$	1000	24-30	Violet	31.66	33.45	33.75	31.74	33.41	34.84
$CaZrS_3$	1150	24	Black	17.7	39.5	42.0	17.54	40.37	42.09

changes in relative intensity with increased temperature of preparation. However, the  $hkl$  reflections remain sharp, and their intensities change only slightly. This is shown in Fig. 1 for the (200) and (002) reflections and by the data in Table 5. The intensities were placed on the same scale by assuming that the intensity of the (110) reflection was constant in all the patterns.

Symmetry requirements permit only variations in the  $x$  and  $z$  parameters of the sulfur atoms and the  $z$  parameters of the other atoms. The intensity data indicate that only changes in the  $z$  parameters need be considered. The origin was fixed at a titanium atom and the effect of variations in the  $z$  parameters of the barium and sulfur atoms upon the calculated intensities was ascertained. The intensity changes listed in Table 5 could not be accounted for, even qualitatively, unless it was assumed that the  $B$  component of the structure factors was greatly decreased. This observation and the broadening of the reflections for which  $l$  is not zero suggests that the structure becomes increasingly disordered with in-

creasing temperature of preparation, *i.e.* with increasing sulfur deficiency.

### Alkaline earth zirconium sulfides

Barium zirconate was allowed to react with  $\text{CS}_2$  in the temperature range 500–1350 °C. for different lengths of time. Below 1000 °C. highly colored products of varying sulfur content were obtained. These products consisted of mixtures of  $\text{BaZrO}_3$ ,  $\text{BaZrS}_3$  and an unidentified phase in the proportions shown in Table 6. The per cent composition of each product was determined by comparing the intensities of several reflections of pure  $\text{BaZrO}_3$  and  $\text{BaZrS}_3$  with their values in the mixtures.

Samples of  $\text{BaZrS}_3$  prepared in the temperature range 950–1200 °C. and 4 to 24 hours reaction time gave identical X-ray diffraction patterns although their sulfur contents varied from 27% to 29.6%. The powder pattern (Table 8) was indexed on the basis of an orthorhombic unit cell with

$$a = 7.037, b = 9.983, c = 7.050 \text{ \AA}.$$

Table 8. Powder patterns of the alkaline earth zirconium sulfides

BaZrS <sub>3</sub>				SrZrS <sub>3</sub>				CaZrS <sub>3</sub>		
<i>hkl</i>	<i>d</i> <sub>o</sub>	<i>d</i> <sub>c</sub>	<i>I</i> / <i>I</i> <sub>0</sub>	<i>hkl</i>	<i>d</i> <sub>o</sub>	<i>d</i> <sub>c</sub>	<i>I</i> / <i>I</i> <sub>0</sub>	<i>d</i> <sub>o</sub>	<i>d</i> <sub>c</sub>	<i>I</i> / <i>I</i> <sub>0</sub>
111	4.461	4.457	4	200	—	—	—	6.53	6.53	14
121	3.528 ±	3.526	100	201	—	—	—	5.83	5.92	12
002		3.525		012	5.84	5.76	5	—	—	—
200		3.519		020	—	4.894	12	4.795	4.790	80
				202	4.901	4.898		—	4.785	
012	3.325	3.325	4	120	4.548 <i>B</i>	4.601	—	—	—	
			300	4.497		4	—	—	—	
			121	4.378		15	4.282	4.282	6	
102	3.149	3.151	2	212	4.380		4.280	—		
201		3.148	004	3.561	3.560	23	3.512	3.512	25	
130	2.998	3.008	5	222	3.463	3.463	100	3.386	3.386	100
112		3.005		400	3.373	3.373	30	3.274	3.267	25
211	2.874 ±	3.003	18	014	3.343	3.346	10	—	—	—
022		2.879		401	—	—	—	—	3.160	3.183
220	2.876	2.876	4	130	—	3.172	—	3.102	—	—
131	2.768	2.767	4	204	3.168	3.150	4	3.090	3.093	4
122	2.668	2.665	3	410	—	3.189	—	3.092	—	—
221		2.663		402	3.050	3.050	3	2.961	2.963	8
040	2.495 ±	2.496	60	214	2.998	2.995	3	2.946	2.944	8
202		2.490		230	2.927	2.937	20	2.868	2.869	22
				412	2.912	2.910	20	—	2.831	80
032	2.419	2.420	5	132	—	2.896	—	2.826	2.837	
212		2.416		024	2.877	2.879	2	—	2.833	
132	2.286	2.288	1	231	—	2.876	—	—	—	—
231		2.287		124	—	—	—	2.763	2.768	20
310	2.234 ±	2.284	1	304	2.791	2.792	10	—	—	—
141		2.232		420	2.781	2.777	10	—	—	—
103	2.228	2.229	1	323	2.712	2.717	12	2.655	2.655	15
222		2.228		232	—	2.715		—	2.656	
301	2.177	2.225	5	115	2.677	2.681	6	—	—	—
113		2.176		500	—	—	—	2.614	2.614	5
311	2.036 ±	2.173	50	224	—	2.649	6	2.596	2.599	14
042		2.037		413	—	2.647		—	—	—
240	2.036 ±	2.036	50	422	2.587	2.587	5	2.520	2.520	8
123		2.036		510	—	—	—	—	2.521	
321	1.918	2.033	1	215	2.532	2.535	7	—	—	—
150		1.921		040	—	2.447	70	2.393	2.395	66
213	1.918	1.918	404	2.447	2.448	—		2.393		
330	1.917	1.917								
312	1.916	1.916								

Table 8 (cont.)

BaZrS <sub>3</sub>				SrZrS <sub>3</sub>				CaZrS <sub>3</sub>		
<i>hkl</i>	<i>d</i> <sub>o</sub>	<i>d</i> <sub>c</sub>	<i>I</i> / <i>I</i> <sub>0</sub>	<i>hkl</i>	<i>d</i> <sub>o</sub>	<i>d</i> <sub>c</sub>	<i>I</i> / <i>I</i> <sub>0</sub>	<i>d</i> <sub>o</sub>	<i>d</i> <sub>c</sub>	<i>I</i> / <i>I</i> <sub>0</sub>
151		1.853		041					2.361	
133	1.855	1.852	3	305				2.361	2.361	12
331		1.850		034		2.405		2.363		
242		1.763		140	2.402	2.408	13			
004	1.763 ≠	1.762	141	2.374						
400		1.759	414	2.373	2.376	13		2.320	2.324	12
052	1.738	1.737	006	2.373						
014		1.735	234	2.266	2.266		2	—	—	
152		1.687		241		2.266		—	—	
251		1.686		206	2.236	2.240	15	2.204	2.204	32
233	1.684	1.685	3	600	—	—		2.173	2.178	4
114		1.685		432	—	—		2.172	2.148	
332		1.684		216		2.183	25	2.144	2.141	55
411		1.682		424	2.183	2.190				
313		1.638		1.638	2	602	2.147	2.145	5	2.082
161		1.578		612	2.089	2.095	5	—	—	
204	1.576 ≠	1.576	15	530		2.080	5	—	—	
323		1.575		342	2.060	2.058	2	—	—	
402		1.574		531		2.058		1.998	2.002	50
252		1.558		226		2.036		2.002	2.001	
034	1.557	1.557	4	007	2.030	2.035	10	2.001	1.975	
214		1.556		603		2.032		1.979	1.979	25
412		1.555		004		2.016		1.984	1.984	
153		1.487		107	2.011	2.012	15	1.980	1.983	
351	1.485	1.486	4	620	—	—		—	—	
333		1.485		117		1.971	20	1.933	1.932	15
044	1.439 ≠	1.439	8	440	1.975	1.981			1.935	
440		1.438		613		1.962		1.908	1.914	
234		1.424		441		1.962	45	1.908	1.915	50
432	1.423	434	1.959	1.959						
115	1.372	343		1.958		1.915		1.916		
171	1.370	1.371	2	050		1.957		1.914		
511		1.367		505		1.595		1.903		
125		1.334		406		—		1.847	2	
361		1.333			1.879		7	1.829	3	
163	1.331 ≠	1.333	18		1.843		2	1.816	3	
244		1.332			1.748		4	1.801	4	
442		1.331			1.727		20	1.755	3	
521		1.330			1.685		4	1.743	3	
135		1.279			1.662 <i>B</i>		5	1.694	20	
353	1.277	1.277	1		1.647		10	1.649	7	
531		1.275			1.545		13	1.630	14	
080	1.245 ≠	1.248	2		1.524		7	1.623	14	
404		1.245			1.496		5	1.609	4	
082		1.176			1.415		5	1.580	5	
280		1.176			1.388		6	1.544	7	
325	1.175 ≠	1.176	8		1.373		2	1.520	7	
363		1.175			1.320		3	1.515	12	
006		1.175			1.316		4	1.495	4	
523	1.173 ≠	1.174	8		1.314		4	1.476	4	
600		1.173			1.297		7	1.471	10	
282	1.116	1.116	4		1.296		8	1.414	12	
206	1.114 ≠	1.114	5		1.293		7	1.392	4	
444		1.114			1.278		7	1.380	4	
602	1.112 ≠	1.113	4		1.275		4	1.374	3	
074		1.108			1.209		6	1.328	4	
036	1.108	1.108	2		1.205		3	1.266	5	
216		1.107			1.201		5	1.262	10	
345		1.089			1.199		3	1.152	4	
183		1.089			1.185		3	1.138	3	
381	1.088	1.088	1		1.183		4	1.134	3	
226		1.088			1.154		4	1.129	3	
543		1.087			1.152		7	1.125	3	
622		1.086			1.148		6	1.116	2	
						1.139		5		

Table 8 (cont.)

BaZrS <sub>3</sub>				SrZrS <sub>3</sub>				CaZrS <sub>3</sub>		
<i>hkl</i>	<i>d<sub>o</sub></i>	<i>d<sub>c</sub></i>	<i>I/I<sub>0</sub></i>	<i>hkl</i>	<i>d<sub>o</sub></i>	<i>d<sub>c</sub></i>	<i>I/I<sub>0</sub></i>	<i>d<sub>o</sub></i>	<i>d<sub>c</sub></i>	<i>I/I<sub>0</sub></i>
373	1.080	1.082	1		1.131		4	1.111		7
191		1.083			1.123		3			
165		1.064			1.093		3			
046	1.061 ≠	1.063	5		1.063		4			
561		1.062			1.090		4			
640		1.061			1.078		4			
092	1.058	1.058	5		1.043		2			
472		1.057			1.029		3			
274		1.057			1.004		3			
454	1.032	1.056	1		1.000		2			
553		1.033			0.9781		3			
265		1.030			0.9761		3			
084	1.017 ≠	1.018	4		0.9748		2			
480		1.018			0.9557		3			
246		1.018			0.9554		2			
642	1.015 ≠	1.016	4		0.9526		2			
292		1.013			0.9503		5			
056		1.012			0.9482		3			
0,10,0	0.9982	0.9983	1		0.9369		3			
175		0.9933			0.9358		3			
193		0.9933			0.9319		2			
391	0.9918	0.9928	2		0.9289		4			
117		0.9921			0.9266		2			
515		0.9921			0.9216		2			
571	0.9784B ≠	0.9918	2		0.9179		3			
1,10,1		0.9789			0.8885		2			
365		0.9785								
284	0.9768B ≠	0.9782	7							
482		0.9778								
127		0.9777								
525	0.9738	0.9777	10							
563		0.9773								
406		0.9770								
604	0.9720	0.9762	8							
256		0.9732								
652		0.9716								
185	0.9271	0.9270	1							
2,10,2		0.9266								
545		0.9259								
266	0.9256	0.9259	1							
581		0.9256								
307		0.9254								
662	0.9224	0.9250	1							
375		0.9226								
393		0.9222								
1,10,3	0.9110	0.9111	4							
3,10,1		0.9109								
446		0.9098								
644	0.9083	0.9092	5							
723		0.9088								
294		0.9070								
3,10,3	0.8558	0.8555	4							
086		0.8554								
565		0.8552								
167	0.8532	0.8552	5							
802		0.8535								
2,11,2		0.8527								
0,12,0	0.8319	0.8319	2							
840		0.8293								
494		0.8285								
575	0.8166	0.8281	3							
717		0.8171								
593		0.8171								
715	0.8094	0.8168	4							
0,12,2		0.8163								
2,12,0		0.8097								
842	0.8072	0.8096	6							
0,11,4		0.8075								
2,12,2		0.8068								
804	0.7888	0.7890	2							
		0.7866								
		0.7870	3							

The unit cell contains 4 BaZrS<sub>3</sub> units based on the density of 4.23 g.cm.<sup>-3</sup> reported by Hahn & Mutschke for their preparation. The systematic absences are (*hk*0) for *h* + *k* odd and (*okl*) for *l* odd. Thus, the space group is either *D*<sub>2h</sub><sup>16</sup>-*Pnmc* or *C*<sub>2v</sub><sup>9</sup>-*Pnc*2<sub>1</sub>. The reflections marked with an † in Table 8 can be indexed on the basis of a tetragonal cell with  $a \cong \frac{1}{2} \sqrt{(2)a_t} \cong \frac{1}{2} \sqrt{(2)c_t}$  and  $c = \frac{1}{2}b_t$ , where the subscript *t* refers to the true unit cell dimensions. These reflections correspond very nearly in position and intensity to those expected for an ideal perovskite structure. Thus, barium zirconium sulfide must have a distorted perovskite structure probably similar to that of CaTiO<sub>3</sub> (Kay & Bailey, 1957) or GdFeO<sub>3</sub> (Geller, 1956). The difference in structure between the barium zirconium sulfide obtained in this study and that reported by Hahn & Mutschke may again be due to a sulfur deficiency resulting from the present method of synthesis.

Attempts to obtain a high temperature modification of BaZrS<sub>3</sub> were unsuccessful. Only mixtures of BaS, ZrS<sub>2</sub> and the BaZrS<sub>3</sub> phase described above were obtained when the reaction was carried out above 1200 °C.

Strontium and calcium zirconium sulfides were prepared from the corresponding ternary oxides at 1000–1200 °C. Below 1000 °C. mixtures of binary oxides and sulfides together with small amounts of ternary oxide and sulfide phases were obtained. Usually, complete reaction was achieved in 24 hours as shown by the analyses in Table 7 and the constancy

of the X-ray patterns on longer reaction. The powder patterns are given in Table 8. It was possible to index the patterns on the basis of orthorhombic unit cells with the following dimensions:

	<i>a</i>	<i>b</i>	<i>c</i>
SrZrS <sub>3</sub>	13.49 Å	9.79 Å	14.23 Å
CaZrS <sub>3</sub>	13.07	9.58	14.05

The unit cells contain 16 molecules based on measured densities of 3.85 g.cm.<sup>-3</sup> for SrZrS<sub>3</sub> and 3.33 g.cm.<sup>-3</sup> for CaZrS<sub>3</sub>. The space groups of these compounds could not be determined because the large unit cell dimensions do not permit unambiguous indexing. However, the similarity of the powder patterns to that of barium zirconium sulfide indicates that SrZrS<sub>3</sub> and CaZrS<sub>3</sub> have highly distorted multiple-cell perovskite structures.

The author wishes to express his sincere thanks to Mr M. J. Franey for his excellent technical assistance, and to the Titanium Alloy control lab for the analytical determinations.

#### References

- CLEARFIELD, A. (1958). *J. Amer. Chem. Soc.* **80**, 6511.  
 GELLER, S. (1956). *J. Chem. Phys.* **24**, 1236.  
 HAHN, H. & MUTSCHKE, U. (1956). *Z. anorg. Chemie*, **288**, 269.  
 KAY, H. F. & BAILEY, P. C. (1957). *Acta Cryst.* **10**, 219.  
 LANDER, J. J. (1951). *Acta Cryst.* **4**, 148.

*Acta Cryst.* (1963). **16**, 142

## A Rapid Method of Obtaining the Third Set of Coordinates in a Crystal Structure Analysis

BY S. G. G. MACDONALD

*Physics Department, University College of the West Indies, Kingston 7, Jamaica*

(Received 10 April 1962)

When two of the coordinates of the atoms in the unit cell of a centrosymmetric crystal have been obtained by finding the projection of the electron density down one of the principal axes, the coordinates in the third direction are sometimes difficult to obtain without a tedious trial-and-error process. The following paper gives a simple method of determining the signs of the larger structure factors for a second projection and hence obtaining the third set of coordinates. The method appears to be useful as long as the number of atoms in the unit cell is no more than fifty to sixty.

### Theory

In a centrosymmetric crystal, any structure factor has the form

$$F(h, k, l) = 2 \sum_{j=1}^{N/2} f_j(h, k, l) \cos 2\pi(hx_j + ky_j + lz_j). \quad (1)$$

In two dimensions and more suitable form, it becomes

$$F(h, k, 0) = 2 \sum_{j=1}^{N/2} f_j(h, k, 0) (\cos 2\pi hx_j \cos 2\pi ky_j - \sin 2\pi hx_j \sin 2\pi ky_j). \quad (2)$$

If the *x<sub>j</sub>* coordinates are as yet undetermined but the *y<sub>j</sub>* coordinates are known,  $|F(h, k, 0)|$  and the factors  $f_j(h, k, 0) \cos 2\pi ky_j$  and  $f_j(h, k, 0) \sin 2\pi ky_j$  are known quantities. A series of simultaneous equations is ob-